HIGHER ORDER THINKING QUESTIONS

CHAPTER :1 (Relations and Functions)

1. If R $= \{(x, y): x + 2y = 8\}$ is a relation on N, write the range of R . **Ans:** $\mathbf{R_R = \{1, 2, 3\}}$

2. Let A = N × N and let * be the binary operation on A, defined by $(a , \mathrm{b}) * (\mathrm{c}, \mathrm{d}) = (a + \mathrm{c}, \mathrm{b} + \mathrm{d})$. Show that * is commutative and associative. Also find the identity element on A if exits. **Ans: NO identity element exists**

3. Consider a function f:$R_+ \rightarrow [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$, show that f is invertible with $f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}$

4. If the function f : R → R be defined by f $(x) = 2x - 3$ and g : R → R by g $(x) = x^3 + 5$, then find the value of (fog) $^{-1}(x)$. **Ans:** $\sqrt[3]{\frac{x-7}{2}}$ **or** $\left(\frac{x-7}{2}\right)^{\frac{1}{3}}$

5. Let A = Q × Q , where Q is the set of all rational numbers, and $*$ be a binary operation defined on A by $(a, \mathrm{b}) * (\mathrm{c}, \mathrm{d}) = (a\mathrm{c},\ \mathrm{b} + a\mathrm{d})$, for all $(a, \mathrm{b})(\mathrm{c}, \mathrm{d}) \in$ A Find :

(i) the identity element in A **Ans:** $(\mathbf{1}, \mathbf{0})$

(ii) the invertible element of A **Ans:** $\left(\frac{1}{a}, \frac{-b}{a}\right)$

6. Let A $= \{1,2,3, \dots, 9\}$ and R be the relation in A X A defined by (a, b) ^{R} (c , d) iff $a + \mathrm{d} = \mathrm{b} + \mathrm{c}$ for all a, b, c ,d ∈ A .Prove that R is an equivalence relation. Also obtain the equivalence class [(2,5)] **Ans:**$[(\mathbf{1}, \mathbf{4}), (\mathbf{2}, \mathbf{5}), (\mathbf{3}, \mathbf{6}), (\mathbf{4}, \mathbf{7}), (\mathbf{5}, \mathbf{8}), (\mathbf{6}, \mathbf{9})]$

7. Let f : N → N be defined by

$$\mathrm{f(n)} = \begin{cases} \mathrm{n} + 1, & \text{if n is odd} \\ \mathrm{n} - 1, & \text{if n is even} \end{cases} ,$$

show that f is bijective. Also find f^{-1}

8. Show that the relation R in the set A $= \{1, 2, 3, 4, 5\}$ given by R $= \{(a, b) : |a - b| \text{is divisible by } 2\}$ is an equivalence relation. Show that all the elements of $\{1, 3, 5\}$ are related to each other and all the elements of $\{2,\ 4\}$ are related to each other, but no element of $\{1, 3, 5\}$ is related to any element of$\{2,\ 4\}$.

9. Let R be a relation on the set A $= \mathrm{N} \times \mathrm{N}$,where N is the set of natural numbers, defined by (x, y) R (u, v) if and only if $x\,\mathrm{v} = y\,\mathrm{u}$. Show that R is an equivalence relation.

10. Show that the relation R in the set $\mathrm{N} \times \mathrm{N}$, defined by (a, b) R (c, d) iff $a^2 + \mathrm{d}^2 = \mathrm{b}^2 + \mathrm{c}^2$

$\forall\, a, \mathrm{b}, \mathrm{c}, \mathrm{d} \in \mathrm{N}$, is an equivalence relation.

11. On the set $\{0, 1, 2, 3, 4, 5, 6\}$, a binary operation $*$ is defined as :

$$a * b = \begin{cases} a + \mathrm{b}, & \text{if } a + \mathrm{b} < 7 \\ a + \mathrm{b} - 7, & \text{if } a + \mathrm{b} \geq 7 \end{cases}$$

Write the operation table of the operation $*$ and prove that zero is the identity for this operation and each element $\boldsymbol{a \neq 0}$ of the set is invertible with $\mathbf{7} - \boldsymbol{a}$ being the inverse of $\boldsymbol{a}$

12. Let f , g : R → R be defined as f $(x) = [x]$,and g $(x) = |x|$, find the values of (i) (gof) $\left(-\frac{5}{3}\right) -$ (fog) $\left(-\frac{5}{3}\right)$(ii)(gof) $\left(\frac{5}{3}\right) -$ (fog) $\left(\frac{5}{3}\right)$(iii)$(\mathrm{f} + 2\mathrm{g})(-1)$ **Ans:1,0,1**

13. Consider the binary operation $*: R \times R \to R$ and $0 : R \times R \to R$ defined as $a * b = |a - b|$ and $a\, 0\, b = a, \forall\, a, b \in R$. Show that $*$ is commutative but not associative 0 is associative but not commutative. Further, show that $\forall\, a, b, c \in R\;\; a * (b\, 0\, c) = (a * b) 0 (a * c)$

14. Let f: R → R be defined as $f(x) = 10x + 7$. Find the function g : R → R such that $(gof) = (fog) = I_R$

15. Let N denote the set of all natural numbers and R be the relation on $N \times N$ defined by $(a, b)\, R\, (c, d)$ iff $a\, d(b + c) = b\, c\, (a + d)$. Show that R is an equivalence relation.

16. Consider a function $f : R_+ \to [-9, \infty)$ given by $f(x) = 5x^2 + 6x - 9$, prove that that f is invertible with $\mathbf{f^{-1}(y) = \frac{\sqrt{54+5y} - 3}{5}}$

CHAPTER :2 (Inverse Trigonometric Functions)

17. Evaluate**:** $\sin\left(2\cos^{-1}\left(-\frac{3}{5}\right)\right)$ **Ans:**$-\frac{24}{25}$

18. Write the principal value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ **Ans:**$\frac{5\pi}{6}$

19. Simplify :$\tan^{-1}\left[2\cos\left\{2\sin^{-1}\left(\frac{1}{2}\right)\right\}\right]$ **Ans:** $\frac{\pi}{4}$

20. Evaluate: $\tan^{-1}(\sqrt{3}) - \cot^{-1}(\sqrt{3})$ **Ans:** $\frac{-\pi}{2}$

21. Find the value of x, if $\sin[\cot^{-1}(x + 1)] = \cos(\tan^{-1} x)$ **Ans:** $-\frac{1}{2}$

22. Solve for x: $\sin^{-1} 6x + \sin^{-1} 6\sqrt{3}x = -\frac{\pi}{2}$ **Ans:**$\boldsymbol{x \pm \frac{1}{12}}$ **but** $\boldsymbol{x = -\frac{1}{12}}$ **why**?

23. Prove that: $\tan\left\{\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right\} + \tan\left\{\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right\} = \frac{2b}{a}$

24. Prove that: $\tan\left\{\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}\right\} = \frac{3-\sqrt{5}}{2}$

25. Prove that: $\sin[\cot^{-1}\{\cos(\tan^{-1} x)\}] = \sqrt{\frac{x^2+1}{x^2+2}}$

26. Prove that: $2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right) = \cos^{-1}\left(\frac{a\cos x + b}{a + b\cos x}\right)$

27. Prove that: $\tan^{-1}\left[\frac{\sqrt{1+\cos x} + \sqrt{1-\cos x}}{\sqrt{1+\cos x} - \sqrt{1-\cos x}}\right] = \frac{\pi}{4} - \frac{x}{2}$ $\quad \pi < x < \frac{3\pi}{2}$

28. Show that:- $\sin^{-1}\frac{12}{13} + \cos^{-1}\frac{4}{5} + \tan^{-1}\frac{63}{16} = \pi$

29. Solve for x : $\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right) + \tan^{-1}\left(\frac{2x}{x^2-1}\right) = \frac{2\pi}{3}$ **Ans: $2 - \sqrt{3}$**

30. If $\tan^{-1}\left(\frac{1}{1+1.2}\right) + \tan^{-1}\left(\frac{1}{1+2.3}\right) +$ ………………… $+\tan^{-1}\left(\frac{1}{1+n(n+1)}\right) = \tan^{-1}\theta$, then find the value of θ **Ans:** $\boldsymbol{\theta = \frac{n}{n+2}}$

31. If $(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$, then find x **Ans: -1**

32. Prove the following : $\cot^{-1}\left(\frac{xy+1}{x-y}\right) + \cot^{-1}\left(\frac{yz-1}{y-z}\right) + \cot^{-1}\left(\frac{zx+1}{z-x}\right) = 0$

33. Prove the following : $\sin\left[\tan^{-1}\left(\frac{1-x^2}{2x}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right] = 1$

34. Prove that : $\tan^{-1}\left(\frac{6x-8x^3}{1-12x^2}\right)-\tan^{-1}\left(\frac{4x}{1-4x^2}\right)=\tan^{-1}2x\,;\ |2x|<\frac{1}{\sqrt{3}}$

35. Solve for x: $\tan^{-1}(x-1)+\tan^{-1}x+\ \tan^{-1}(x+1)=\tan^{-1}3x$ **Ans:** $\boldsymbol{x=\pm\frac{1}{2}}$

36. Solve for x: $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$ **Ans: 0**

CHAPTER :4 (Determinants)

37. If A is a square matrix of order 3 and $|3\,A|=K|A|$, then write the value of K **Ans: 27**

38. If A is a square matrix of order 3 such that $|\text{adj } A|=225$, find $|A'|$ **Ans:$\pm$15**

39. Using properties of determinant prove that:

$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$$

40. If $a\neq b\neq c$ and $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}=0$, then using properties of determinants, prove that $a+b+c=0$

41. Using properties of determinants , prove that

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$$

42. If A= $\begin{bmatrix} 2 & 3 & 1 \\ -3 & 2 & 1 \\ 5 & -4 & -2 \end{bmatrix}$, find $A^{-1}$and hence solve the given equations:

$2x-3y+5z=11,\ 3x+2y-4z=-5,\ x+y-2z=-3$ **Ans:** $\boldsymbol{x=1, y=2, z=3}$

43. Let A = $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, B = $\begin{bmatrix} 4 & -6 \\ -2 & 4 \end{bmatrix}$. Compute AB. Hence, solve the following system of equations: $2x+y=4,\quad 3x+2y=1.$ **Ans:** $\boldsymbol{x=7, y=-10}$

44. Find the product AB, where A = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$,B = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ and use it to solve the equations: $x-y=3, 2x+3y+4z=17, y+2z=7$ **Ans:** $\boldsymbol{x=2, y=-1, z=4}$

45. On her birthday Seema decided to donate some money to children of an orphanage home. If there were 8 children less, everyone would have got Rs. 10 more. However, if there were 16 children more, every one would have got Rs. 10 less. Using matrix method, find the number of children and the amount distributed by Seema. What values are reflected by Seema's decision? **Ans: 32,Rs.960**

CHAPTER: 5 (Continuity and Differentiability)

46. Find the derivative of f $(e^{\tan x})$ w. r. to $x=0$. It is given that $f'(1)=5$ **Ans:5**

47. Let f $(x)=x-|x-x^2|, x\in[-1,1]$. Find the point of discontinuity,(if any) of this function on$[-1,1]$

48. If $x = \cos t(3 - 2\cos^2 t)$ and $y = \sin t(3 - 2\sin^2 t)$, find the value of $\frac{dy}{dx}$ at t = $\frac{\pi}{4}$ **Ans:1**

49. If $x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$ and $y = a\sin\theta$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$ **Ans:**$\frac{2\sqrt{2}}{a}$

50. Let $f(x) = \begin{cases} \frac{1-\sin^3 x}{3\cos^2 x}, & \text{if } x < \frac{\pi}{2} \\ a, & \text{if } x = \frac{\pi}{2} \\ \frac{b(1-\sin x)}{(\pi-2x)^2}, & \text{if } x > \frac{\pi}{2} \end{cases}$, if $f(x)$ is continuous at $x = \frac{\pi}{2}$, find a and b **Ans:** $a = \frac{1}{2}, b = 4$

51. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, show that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

52. Find the value of 'a' for which the function f defined as : $f(x) = \begin{cases} a\sin\frac{\pi}{2}(x+1), & x \le 0 \\ \frac{\tan x - \sin x}{x^3}, & x > 0 \end{cases}$ is continuous at $x = 0$ **Ans:** $a = \frac{1}{2}$

53. Show that the function f defined as follows is continuous at $x = 2$, but not differentiable there at :

$$f(x) = \begin{cases} 3x - 2, & 0 < x \le 1 \\ 2x^2 - x, & 1 < x \le 2 \\ 5x - 4, & x > 2 \end{cases}$$

54. The function $f(x)$ is defined as $f(x) = \begin{cases} x^2 + ax + b, & 0 \le x < 2 \\ 3x + 2, & 2 \le x \le 4 \\ 2ax + 5b, & 4 < x \le 8 \end{cases}$

If $f(x)$ is continuous on $[0, 8]$, find the values of 'a' and 'b' **Ans:** $a = 3, b = -2$

55. If $x = 3\sin t - \sin 3t$ and $y = 3\cos t - \cos 3t$, find $\frac{d^2y}{dx^2}$ at t = $\frac{\pi}{3}$ Ans:$\frac{-16}{27}$

56. If $x = a\sin 2t\,(1 + \cos 2t)$ and $y = b\cos 2t\,(1 - \cos 2t)$, show that at $t = \frac{\pi}{4}$, $\left(\frac{dy}{dx}\right) = \frac{b}{a}$

57. If $x = \sin t, y = \sin kt$, show that: $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + k^2 y = 0$

58. If function $f(x) = |x - 3| + |x - 4|$, then show that $f(x)$ is not differential at $x = 3$ and $x = 4$.

59. Find a, b, c if the given function $f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x}, & x < 0 \\ c, & x = 0 \\ \frac{\sqrt{x + bx^2} - \sqrt{x}}{bx\sqrt{x}}, & x > 0 \end{cases}$, is continuous at $x = 0$

Ans: $a = -\frac{3}{2}, c = \frac{1}{2}$ and b is any non-zero real number

60. If the following function is differentiable at $x = 2$, then find the values of a and b

$$f(x) = \begin{cases} x^2, & \text{if } x \le 2 \\ ax + b, & \text{if } x \ge 2 \end{cases}$$

Ans: $a = 4, y = -4$

61. If $y = \sin^{-1}\left[\frac{5x + 12\sqrt{1-x^2}}{13}\right]$, find $\frac{dy}{dx}$. **Ans:**$\frac{-1}{\sqrt{1-x^2}}$

62. It is given that for the function $f(x) = x^3 + bx^2 + ax + 5$ on $[1, 3]$, Rolle's theorem holds with $c = 2 + \frac{1}{\sqrt{3}}$. Find the values of a and b **Ans:** $a = 11, b = -6$

63. For the function $(x) = x^3 - 6x^2 + ax + \text{b}$, it is given that f (1) =f (3) = 0. Find the values of a and b, and hence verify Roll′ s Theorem on $[1, 3]$ **Ans :** $\boldsymbol{a = 11, b = -6}$ **,** $\mathbf{c = 2 \pm \frac{1}{\sqrt{3}}}$

64. Differentiate the following with respect to x : $\sin^{-1}\left(\frac{2^{x+1}\,.\,3^x}{1+(36)^x}\right)$ **Ans:**$\frac{\mathbf{2^{x+1}.3^x \log 6}}{\mathbf{1+36^x}}$

CHAPTER: 6(Application of Derivatives)

65. Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$

66. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin. **Ans:** $\mathbf{(0, 0)(1, 2)(-1, -2)}$

67. Using derivative, find the approximate percentage - increase in the area of a circle if its radius is increased by 2%**.** **Ans:4%.**

68. Find the equations of the tangents to the curve $3x^2 - y^2 = 8$, which pass through the point $\left(\frac{4}{3}, 0\right)$ **Ans:** $\mathbf{-2y + 2 = 0}$ **,** $\mathbf{3x + 2y + 2 = 0}$

69. Find the intervals in which the function f, given by f $(x) = \sin x + \cos x$, $0 \leq x \leq 2\pi$

Is (i) strictly increasing (ii) strictly decreasing. **Ans:** $\left[\mathbf{0}, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}\right]$ **and** $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

70. Find the equation of the normal to the curve $x^2 = 4y$, which passes through the point $(1,2)$

Ans: $\boldsymbol{x + y = 3}$

71. Find the intervals in which the following function is strictly increasing or strictly decreasing. f $(x) = (x-1)^3.\,(x+2)^2$. Also, find the points of local maximum and local minimum, if any. **Ans: f is S.I. in** $(-\infty, -2]$ **and** $[\frac{-4}{5}, \infty)$ **f is S.D. in** $[-2, \frac{-4}{5}]$ **,−2 (local maximum),** $\frac{-4}{5}$**(local minimum)**

72. If the function f $(x) = 2x^3 - 9\,\text{m}x^2 + 12\,\text{m}^2 x + 1$, where $\text{m} > 0$ attains its maximum and minimum at p and q respectively such that p^2= q, then find the value of m. **Ans: 2**

73. Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ with its vertex at one end of the major axis. **Ans:** $\mathbf{9\sqrt{3}}$

74. A jet of enemy is flying along the curve $y = x^2 + 2$ and a soldier is placed at a point (3, 2). Find the minimum distance between the soldier and the jet**.** **Ans: :** $\sqrt{\mathbf{5}}$

75. If the sum of the lengths of the hypotenuse and a side of a right triangle is given, show that the area of the triangle is maximum, when the angle between them is 60^0

76. An open box, with a square base is to be made out of a given quantity of metal sheet of area c^2. Show that the maximum volume of the box is $\frac{C^3}{6\sqrt{3}}$.

77. A manufacturer can sell x items at a price of rupees $\left(5 - \frac{x}{100}\right)$ each. The cost price is Rs. $\left(\frac{x}{5} + 500\right)$.Find the number of items he should sell to earn maximum profit. **Ans : Rs 240**

78. Prove that the radius of the right circular cylinder of greatest curved surface which can be inscribed in a given cone is half of that of the given cone**.**

79. A farmer wants to construct a circular garden and a square garden in his field. He wants to keep the sum of their perimeters 600 m. Prove that the sum of their areas is the least, when the side of the square garden is double the radius of the circular garden. Do you think that a good planning can save energy, time and money? **Ans:**$\boldsymbol{r = \frac{300}{\pi+4} m, x = \frac{600}{\pi+4} m}$

CHAPTER :7 (Integrals)

80. Find : $\int \frac{x\,dx}{1+x\tan x}$ **Ans:** $\log|\cos x + x\sin x| + c$

81. Find: $\int \frac{x^4}{(x-1)(x^2+1)}dx$ **Ans:** $\frac{x^2}{2} + x + \frac{1}{2}\log|x-1| - \frac{1}{4}\log|x^2+1| - \frac{1}{2}\tan^{-1}x + c$

82. Evaluate: $\int \frac{1+\sin 2x}{1+\cos 2x}e^{2x}dx$ **Ans:** $\frac{1}{2}e^{2x}\tan x + c$

83. Evaluate: $\int \frac{dx}{x(x^n+1)}$ **Ans:** $\frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right| + c$

84. Find $\int \frac{x^2+1}{x^4+1}dx$ **Ans:** $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}\,x}\right) + c$

85. Evaluate: $\int \frac{(x-4)}{(x-2)^3}e^x\,dx$ **Ans:** $e^x\frac{1}{(x-2)^2} + c$

86. Find: $\int \frac{1-\cos x}{\cos x\,(1+\cos x)}dx$ **Ans:** $\log|\sec x + \tan x| - 2\tan\frac{x}{2} + c$

87. Evaluate: $\int_0^{\frac{3}{2}}|x.\cos(\pi x)|\,dx$ **Ans:** $\frac{5}{2\pi} - \frac{1}{\pi^2}$

88. Evaluate: $\int_{-1}^{\frac{3}{2}}|x\sin(\pi x)|\,dx$ **Ans:** $\frac{3}{\pi} + \frac{1}{\pi^2}$

89. Evaluate: $\int_{-1}^{1}|x\cos\pi x|\,dx$ **Ans:** $\frac{2}{\pi}$

90. Evaluate: $\int \frac{1}{\cos^4 x+\sin^4 x}dx$ **Ans: :** $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan^2 x-1}{\sqrt{2}\tan x}\right) + c$

91. Evaluate: $\int \frac{1}{\sin^4 x+\sin^2 x\cos^2 x+\cos^4 x}dx$ **Ans: :** $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{\tan^2 x-1}{\sqrt{3}\tan x}\right) + c$

92. Evaluate: $\int\left(\sqrt{\cot x} + \sqrt{\tan x}\right)dx$ **Ans:** $\sqrt{2}\tan^{-1}\left(\frac{\tan x-1}{\sqrt{2\tan x}}\right) + c$ **OR** $\sqrt{2}\sin^{-1}(\sin x - \cos x) + c$

93. Evaluate: $\int_0^1 \frac{\log(1+x)}{1+x^2}dx$ **Ans:** $\frac{\pi}{8}\log 2$

94. Find $\int_0^2 (x^2 + e^{2x+1})\,dx$ as the limit of a sum. **Ans:** $\frac{8}{3} + \frac{e^5-e}{2}$

95. Evaluate : $\int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x}$ **Ans:** $\frac{\pi^2}{2ab}$

96. Evaluate: $\int_0^{\frac{\pi}{2}}(2\log\sin x - \log\sin 2x)\,dx$ **Ans:** $-\frac{\pi}{2}\log 2$

97. Evaluate : $\int \frac{\sqrt{x^2+1}[\log(x^2+1)-2\log x]}{x^4}dx$ **Ans:** $-\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{\frac{3}{2}}\left[\log\left(1+\frac{1}{x^2}\right) - \frac{2}{3}\right] + c$

98. Evaluate: $\int_1^4 [|x-1| + |x-2| + |x-3|]\,dx$ **Ans:** $\frac{19}{2}$

99. Evaluate: $\int_0^{\frac{\pi}{4}}\log(1+\tan x)\,dx$ **Ans:** $\frac{\pi}{8}\log 2$

100. Evaluate: $\int_0^{\pi}\frac{1}{5+4\cos x}\,dx$ **Ans:** $\frac{\pi}{3}$

101. Evaluate : $\int_0^{\frac{\pi}{2}}\log\sin x\,dx$ **Ans:** $-\frac{\pi}{2}\log 2$

102. Evaluate : $\int \frac{x+\sin x}{1+\cos x}dx$ **Ans:** $x\tan\frac{x}{2} + c$

103. Evaluate : $\int \frac{x-\sin x}{1-\cos x}dx$ **Ans:** $-x\cot\frac{x}{2} + c$

104. Evaluate: $\int \frac{1}{\sin x - \sin 2x}dx$ **Ans:** $-\frac{1}{2}\log|1-\cos x| - \frac{1}{6}\log|1+\cos x| + \frac{2}{3}\log|1-2\cos x| + c$

105. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$, **Ans:** $\frac{\pi^2}{16}$

106. Evaluate the following definite integral: $\int_{-\pi}^{\pi} \frac{2x\,(1+\sin x)}{1+\cos^2 x} dx$ **Ans:** π^2

107. Evaluate the following definite integral: $\int_0^{2\pi} \frac{1}{1+e^{\sin x}} \mathrm{d}x$ **Ans:** π

CHAPTER :8 (Application of Integrals)

108. Using integration, find the area of the region $\{(x,y): |x-1| \le y \le \sqrt{5-x^2}\}$ **Ans:** $(\frac{5\pi}{4} - \frac{1}{2})$**sq.units**

109. Using integration, find the area of the region $\{(x,y): |x+2| \le y \le \sqrt{20-x^2}\}$ **Ans:** $(5\pi - 2)$**sq.units**

110. Using integration, find the area of the region $\{(x,y): 9x^2 + y^2 \le 36 \text{ and } 3x + y \ge 6\}$ **Ans:** $(3\pi - 6)$**sq.units**

111. Sketch the graph of $y = |x+3|$ and hence evaluate $\int_{-6}^{0} |x+3| dx$ **Ans: 9sq.units**

112. Using integration, Find the area bounded by the curves $y = |x-1|$ and $y = 3 - |x|$ **Ans: 4**

113. Find the area bounded by the curves $(x-1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$ **Ans:** $\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$ **sq. unit**

114. Find the area of the circle $x^2 + y^2 = 16$ exterior to the parabola $y^2 = 6x$ **Ans :** $\frac{32\pi - 4\sqrt{3}}{3}$**sq.units**

115. Find the area of the circle $4x^2 + 4y^2 = 9$,which is interior to the parabola $x^2 = 4y$ **Ans:** $\frac{\sqrt{2}}{6} + \frac{9}{4} \sin^{-1} \frac{2\sqrt{2}}{3}$

116. Find the area of the region bounded between the parabola the curves $4y = 3x^2$ and the line $3x - 2y + 12 = 0$ **Ans: 27 sq.units**

117. Using integration, find the area bounded by the tangent to the curve $4y = x^2$ at the point $(2,\ 1)$ and the lines whose equations are $x = 2y$ and $x = 3y - 3$ **Ans: 1 sq units**

118. Using integration, find the area of the triangle formed by positive x-axis and tangent and the normal to the circle $x^2 + y^2 = 4$ at the point $(1,\ \sqrt{3})$. **Ans:** $2\sqrt{3}$ **sq units**

119. Using integration, find the area of the triangle formed by negative x- axis and tangent and normal to the circle $x^2 + y^2 = 9$ at $(-1,\ 2\sqrt{2})$. **Ans:** $9\sqrt{2} sq.units$

CHAPTER : 9 (Differential Equations)

120. Find particular solution of the differential equation: $\frac{dy}{dx} = 1 + x + y + xy$, given that when $y(1) = 0$ **Ans:** $\log|1+y| = x + \frac{x^2}{2} - \frac{3}{2}$

121. Find the particular solution of the differential equation : $\log\left(\frac{dy}{dx}\right) = 3x + 4y$,given that $y = 0$ when $x = 0$. **Ans:** $4e^{3x} + 3e^{-4y} - 7 = 0$

122. Find the particular solution of differential equation $\frac{dy}{dx} + y \cot x = 2x + x^2 \cot x$, given that $y = 0$ when $x = \frac{\pi}{2}$ **Ans:** $y\sin x - x^2 \sin x + \frac{\pi^2}{4} = 0$

123. Obtain the differential equation of all circles of radius r. **Ans:** $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = r^2 (\frac{d^2y}{dx^2})^2$

124. Obtain the differential equation of all circles of radius 3. **Ans:** $\left[\mathbf{1}+\left(\frac{dy}{dx}\right)^2\right]^3=\mathbf{9}\left(\frac{d^2y}{dx^2}\right)^2$

125. Solve the following differential equation, given that $y=0$, when $x=\frac{\pi}{4}$: $\sin 2x\frac{dy}{dx}-y=\tan x$
Ans: $\boldsymbol{y=tan\,x-\sqrt{tan\,x}}$

126. Find a particular solution of differential equation $(x-y)(dx+dy)=dx-dy$, given that $y=-1$, when $x=0$ **Ans:** $\boldsymbol{x+y=\log|x-y|-1}$

127. Solve the deferential equation:-$\left\{x\cos\left(\frac{y}{x}\right)+y\sin\frac{y}{x}\right\}ydx=\left\{y\sin\frac{y}{x}-x\cos\frac{y}{x}\right\}xdy$**Ans:**$\boldsymbol{xycos\frac{y}{x}=A}$

128. Form the differential equation of family of circles in the second quadrant which touch the coordinate axes. ***Ans***: $\boldsymbol{(x+y)^2[(y')^2+1]=[x+yy']^2}$

129. Find particular solution of $(x+1)\frac{dy}{dx}=2e^{-y}-1$, given that $y=0, x=0$,**Ans:** $\boldsymbol{(x+1)(2-e^y)=1}$

130. $(x^2+y^2)dy=xy\,dx$. If $y(1)=1$ and $y(x_0)=e$, then find the value of x_0**Ans:** $\boldsymbol{x_0=\sqrt{3}e}$

131. Find the particular solution of the differential equation:$\frac{dy}{dx}+y\tan x=3x^2+x^3\tan x, x\neq\frac{\pi}{2}$, given that $y=0$ when $x=\frac{\pi}{3}$ **Ans:** $\boldsymbol{y=x^3-\frac{2\pi^3}{27}cosx}$

132. Find the differential equation for all straight lines, which are at a unit distance from the origin. **Ans:** $\boldsymbol{(1-x^2)\left(\frac{dy}{dx}\right)^2+2xy\frac{dy}{dx}+(1-y^2)=0}$

CHAPTER : 10 (Vectors)

133. If $|\vec{a}|=13, |\vec{b}|=5$ and $\vec{a}.\vec{b}=60$, then find $|\vec{a}\times\vec{b}|$ **Ans: 25**

134. Find the area of the parallelogram, whose diagonals are $\vec{d}_1=5\hat{\imath}$and $\vec{d}_2=2\hat{\jmath}$ **Ans:5sq.units.**

135. If $\vec{a}$ and $\vec{b}$ are perpendicular vectors, $|\vec{a}+\vec{b}|=13$ and $|\vec{a}|=5$, find the value of$|\vec{b}|$. **Ans:12**

136. If $|\vec{a}|=a$, then find the value of the following : **Ans:** $\boldsymbol{2a^2}$

$$|\vec{a}\times\hat{\imath}|^2+|\vec{a}\times\hat{\jmath}|^2+|\vec{a}\times\hat{k}|^2$$

137. Find a unit vector perpendicular to the plane of the triangle ABC, where the coordinates of its vertices are A $(3,-1,2)$, B $(1,-1,-3)$ and C $(4,-3,1)$. **Ans:** $\boldsymbol{\frac{1}{\sqrt{165}}(10\hat{\imath}+7\hat{\jmath}-4\hat{k})}$

138. The scalar product of the vector$\vec{a}=\hat{\imath}+\hat{\jmath}+\hat{k}$ with a unit vector along the sum of the vectors $\vec{b}=2\hat{\imath}+4\hat{\jmath}-5\hat{k}$ and $\vec{c}=\mu\hat{\imath}+2\hat{\jmath}+3\hat{k}$ is equal to one .Find the value of μ and hence find the unit vector along $\vec{b}+\vec{c}$ **Ans:**$\boldsymbol{\mu=1,\frac{1}{7}(3\hat{\imath}+6\hat{\jmath}-2\hat{k})}$

139. If $\vec{a}=\hat{\imath}+\hat{\jmath}+\hat{k}$ and $\vec{b}=\hat{\jmath}-\hat{k}$, find a vector $\vec{c}$ such that $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}.\vec{c}=3$ **Ans**: $\boldsymbol{\vec{c}=\left(\frac{5}{3}\hat{\imath}+\frac{2}{3}\hat{\jmath}+\frac{2}{3}\hat{k}\right)}$

140. Express the vector $\vec{a}=5\hat{\imath}-2\hat{\jmath}+5\hat{k}$ as the sum of two vectors such that one is parallel to the vector $\vec{b}=3\hat{\jmath}+\hat{k}$ and other is perpendicular to$\vec{b}$ **Ans**: $\boldsymbol{\vec{c}=\frac{-3}{10}\hat{\imath}-\frac{1}{10}\hat{k}}$&$\boldsymbol{\vec{d}=5\hat{\imath}-\frac{17}{10}\hat{\jmath}+\frac{51}{10}\hat{k}}$

141. If $\vec{a}+\vec{b}+\vec{c}=0$ and $|\vec{a}|=3$, $|\vec{b}|=5$ and $|\vec{c}|=7$, show that the angle between $\vec{a}$ and $\vec{b}$is$\frac{\pi}{3}$

142. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors such that $|\vec{a}| = 1, |\vec{b}| = 4, |\vec{c}| = 2$ and $\vec{a} + \vec{b} + \vec{c} = 0$, then find the value of μ, if $\mu = \vec{a}\vec{b} + \vec{b}\vec{c} + \vec{c}\vec{a}$ **Ans:** $-\frac{21}{2}$

143. Let If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors such that $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{c}| = 5$ and each one of them is perpendicular to the sum of other two, find $|\vec{a} + \vec{b} + \vec{c}|$ **Ans:** $5\sqrt{2}$

144. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, prove that $\vec{a} - \vec{d}$ is parallel to $\vec{b} - \vec{c}$, provided $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$

145. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$, then prove that (i) $\vec{a} = \pm 2(\vec{b} \times \vec{c})$ (ii) $[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}] = \pm 1$

CHAPTER :11 (Three Dimensional Geometry)

146. From the point P(a, b, c), perpendiculars PL and PM are drawn to YZ and ZX planes respectively. Find the equation of the plane OLM. **Ans:** $\boldsymbol{bcx + acy - abz = 0}$

147. A line passes through $(2, -1, 3)$ and is perpendicular to the lines $\vec{r} = (\hat{\imath} + \hat{\jmath} - \hat{k}) + \lambda(2\hat{\imath} - 2\hat{\jmath} + \hat{k})$ and $\vec{r} = (2\hat{\imath} - \hat{\jmath} - 3\hat{k}) + \mu(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$. Obtain its equation in vector and Cartesian form. **Ans:** $\boldsymbol{\vec{r} = (2\hat{\imath} - \hat{\jmath} + 3\hat{k}) + \lambda(2\hat{\imath} + \hat{\jmath} - 2\hat{k})}$

148. Find the equation of the line through the point $(1, -1, 1)$ and perpendicular to the lines joining the points$(4,3,2), (1, -1, 0)$and $(1, 2, -1), (2, 1, 1)$ **Ans:** $\boldsymbol{\vec{r} = \hat{\imath} - \hat{\jmath} + \hat{k} + \lambda(10\,\hat{\imath} - 4\hat{\jmath} - 7\hat{k})}$

149. Find the vector equation of the plane through the points $(-1, 1, 1)$ and $(1, -1, 1))$ and perpendicular to the plane $x + 2y + 2z - 5 = 0$ **Ans:** $\boldsymbol{\vec{r}.(2\hat{\imath} - 2\hat{\jmath} + 3\hat{k}) + 1 = 0}$,

150. Find the image of the point P $(3, 5, 3)$in the line $\vec{r} = (\hat{\jmath} + 2\hat{k}) + \lambda(\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$ **Ans:** $\boldsymbol{(-1, 1, 7)}$

151. Find the coordinates of the image of the point $(1, 3, 4)$ in the given plane $2x - y + z + 3 = 0$ **Ans:** $\boldsymbol{(-3, 5, 2)}$

152. Find the coordinates of the foot of the perpendicular drawn from the point A $(-1, 8, 4)$to the line joining the points B $(0, -1, 3)$ and C $(2, -3, -1)$. Also find the length this perpendicular. **Ans: (-2,1,7)&**$\boldsymbol{\sqrt{59}}$

153. Find the vector and the Cartesian equations of the plane which bisects the line joining the points $(3, -2, 1)$ and $(1, 4, -3)$ at right angles. **Ans:** $\boldsymbol{x - 3y + 2z + 3 = 0}$**,&**$\boldsymbol{\vec{r}.(\hat{\imath} - 3\hat{\jmath} + 2\hat{k}) + 3 = 0}$

154. Show that the following two lines are coplanar:

$$\frac{x-a+d}{\alpha-\delta} = \frac{y-a}{\alpha} = \frac{z-a-d}{\alpha+\delta} \quad \text{and} \quad \frac{x-b+c}{\beta-\gamma} = \frac{y-b}{\beta} = \frac{z-b-c}{\beta+\gamma}$$

155. Find the acute angle between the plane $5x - 4y + 7z - 13 = 0$ and the y − axis **Ans:** $\boldsymbol{sin^{-1}\left(\frac{2}{15}\sqrt{10}\right)}$

156. Find the direction ratios of the normal to the plane, which passes through the points $(1, 0, 0)$ and $(0, 1, 0)$ and makes angle $\frac{\pi}{4}$ with the plane $x + y = 3$. Also find the equation of the plane. **Ans:** $< 1, 1, \pm\sqrt{2} >$, $\boldsymbol{x + y \pm \sqrt{2}z = 1}$

157. Show that the lines $\vec{r} = (-3\,\hat{\imath} + \hat{\jmath} + 5\,\hat{k}) + \lambda(-3\hat{\imath} + \hat{\jmath} + 5\hat{k})$ and $\vec{r} = (-\hat{\imath} + 2\hat{\jmath} + 5\hat{k}) + \mu(-\hat{\imath} + 2\hat{\jmath} + 5\hat{k})$ are coplanar. Also find the vector equation of the plane containing these lines. **Ans:** $\boldsymbol{x - 2y + z = 0}$ **or** $\boldsymbol{\vec{r}.(\hat{\imath} - 2\hat{\jmath} + \hat{k}) = 0}$

158. Find the equation of the plane which contains the line of intersection of the planes $\vec{r}.(\hat{\imath}+2\hat{\jmath}+3\hat{k})-4=0$ and $\vec{r}.(2\hat{\imath}+\hat{\jmath}-\hat{k})+5=0$ and is perpendicular to the plane $\vec{r}.(5\hat{\imath}+3\hat{\jmath}-6\hat{k})+8=0$ **Ans:** $\mathbf{33x+45y+50z-41=0}$

159. Find the shortest distance between the lines whose vector equations are: $\vec{r}=(1-t)\hat{\imath}+(t-2)\hat{\jmath}+(3-2t)\hat{k}$ and $\vec{r}=(s+1)\hat{\imath}+(2s-1)\hat{\jmath}-(2s+1)\hat{k}$ **Ans:** $\mathbf{\frac{8\sqrt{29}}{29}}$

160. Show that the lines: $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}$ and $\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$ intersect each other.

161. Find the vector and Cartesian equation of the plane containing the lines:
$\vec{r}=2\hat{\imath}+\hat{\jmath}-3\hat{k}+\lambda(\hat{\imath}+2\hat{\jmath}+5\hat{k})$ and $\vec{r}=3\hat{\imath}+3\hat{\jmath}+2\hat{k}+\mu(3\hat{\imath}-2\hat{\jmath}+5\hat{k})$.Also show that the line $\vec{r}=2\hat{\imath}+5\hat{\jmath}+2\hat{k}+p(3\hat{\imath}-2\hat{\jmath}+5\hat{k})$ lies in the plane.
Ans: $\mathbf{10x+5y-4z-37=0,\ \vec{r}.(10\hat{\imath}+5\hat{\jmath}-4\hat{k})-37=0}$

162. If the angle between the line $x=\frac{y-1}{2}=\frac{z-3}{\lambda}$ and the plane $x+2y+3z=4$ is $\cos^{-1}\sqrt{\frac{5}{14}}$, then find the value of $\boldsymbol{\lambda}$. **Ans:** $\boldsymbol{\lambda=\frac{2}{3}}$

163. Find the equation of the plane (s) passing through the intersection of the planes $x+3y+6=0$ and $3x-y-4z=0$ and whose perpendicular distance from the origin is unity. **Ans:** $\mathbf{2x+y-2z+3=0}$,& $\mathbf{x-2y-2z+3=0}$

164. Find whether the lines $\vec{r}=(\hat{\imath}-\hat{\jmath}-\hat{k})+\lambda(2\hat{\imath}+\hat{\jmath})$ and $\vec{r}=(2\hat{\imath}-\hat{\jmath})+\mu(\hat{\imath}+\hat{\jmath}-\hat{k})$ intersect or not .If intersecting, find their point of intersection. **Ans:** $\mathbf{(3,0,-1)}$

165. Show that the four points $(0,-1,-1)$, $(4,5,1)$, $(3,9,4)$ and $(-4,4,4)$ are coplanar. Also find the equation of the plane containing them. **Ans**: $5x-7y+11z+4=0$

166. Find the distance of the point $(2,3,4)$ from the line $\frac{x+3}{3}=\frac{y-3}{6}=\frac{z}{2}$ measured parallel to the plane $3x+2y+2z-5=0$ **Ans**: $\mathbf{\sqrt{33}}$

167. Find the distance of the point $3\hat{\imath}-2\hat{\jmath}+\hat{k}$ from the plane $3x+y-z+2=0$ measured parallel to the line $\frac{x-1}{2}=\frac{y+2}{-3}=\frac{z-1}{1}$. Also find the foot of the perpendicular from the given point upon the given plane. **Ans:** $\mathbf{4\sqrt{14}}$ **units**, $\mathbf{\left(\frac{-3}{7},\frac{-22}{7},\frac{15}{7}\right)}$

CHAPTER: 12 (Linear Programming Problems)

168. The postmaster of a local post office wishes to hire extra helpers during the Deepawali season, because of a large increase in the volume of mail handing and delivery. Because of the limited office space and the budgetary conditions, the number of temporary helpers must not exceed 10. According to the past experience, a man can handle 300 letters and 80 packages per day, on the average, and a woman can handle 400 letters and 50 packages per day. The postmaster believes that the daily volume of extra mail and packages will be no less than 3400 and 680 respectively. A man receives Rs. 225 a day and a woman receives Rs. 200 a day. How many men and women helpers should be hired to keep the pay- roll at a minimum? Formulate an LPP and solve it graphically. **Ans:** $\mathbf{6,4}$

169. A man rides his motorcycle at the speed of 50 km/h.He has to spend Rs.2 per km on petrol. If he rides it at a faster speed of 80 km/h,he petrol cast increases to Rs.3per km. He has at most Rs.120 to spend on petrol and one hour's time. Using LPP, find the maximum distance he can travel. **Ans:** $\mathbf{54\frac{2}{7}km}$ **at** $\mathbf{\left(\frac{300}{7},\frac{80}{7}\right)}$

170. An aeroplane can carry a maximum of 200 passengers. A profit of Rs 1000 is made on each executive class ticket and a profit of Rs 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. What is maximum profit? Make it as LPP and solve it graphically. Write one value that travelling learns to us **Ans: Rs 1,36,000.at (4,160)**

171. A dietician wishes to mix two types of food P and Q in such a way that the vitamin contents of mixture contains at least 8 units of vitamin A and 11 units of vitamin B. Food P contains 3 units/kg of vitamin A and 5 units/kg of vitamin B. While food Q contains 4 units/kg of vitamin A and 2 units/kg of vitamin B. It costs Rs 60 per kg to purchase food P and Rs 80 per kg to purchase food Q. Determine the minimum cost of such a mixture. Formulate the above as a LPP and solve it graphically. Write one more vitamin that the dietician should mix in the food. Why? **Ans: Rs 160 at ($\frac{8}{3}$,0) and (2,$\frac{1}{2}$)**

172. A dietician wants to develop a special diet using two foods X and Y. Each packet (contains 30 g) of food X contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of same quantity of food Y contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, at least 460 units of iron and at most 300 units of cholesterol. Make an LPP to find how many packets of each food should be used to minimize the amount of vitamin A in the diet, and solve it graphically. **Ans: vitamin A=150 Units X=15 Pkts,Y=20 Pkts**

173. One kind of cake requires 200 g of floor and 25 g of fat, and another kind of cake requires 100 g of floor and 50 g of fat. Find the maximum number of cakes which can be made from 5 kg of floor and 1 kg of fat assuming that there is no shortage of the ingredients used in making the cakes. **Ans: 30,10**

174. A library has to accommodate two different types of books on a shelf. The books are 6 cm and 4 cm thick and weigh 1 kg and $1\frac{1}{2}$ kg each respectively. The shelf is 96 cm long and at most can support a weight of 21 kg. How should the shelf be filled with the books of two types in order to include the greatest number of books? Make it as an L.P.P. and solve it graphically. **Ans: 18 books at 12 and 6**

CHAPTER: 13 (Probability)

175. In 3 trials of a binomial distribution, the probability of exactly 2 successes is 9 times the probability of 3 successes. Find the probability of success in each trial. **Ans:** $p = \frac{1}{4}$

176. An urn contains 3 red and 5 black balls. A ball is drawn at random; its colour is noted and returned to the urn. Moreover, 2 additional balls of the colour noted down, are put in the urn and then two balls are drawn at random (without replacement) from the urn. Find the probability that both the balls drawn are of red colour. **Ans:** $\frac{1}{8}$

177. An experiment succeeds thrice as often as it fails .Find the probability that in the next five trials, there will be at least 3 successes. **Ans:**$\frac{51}{58}$

178. Consider the experiment of tossing a coin. If the coin shows tail, toss it again but if it shows head, then throw a die. Find the conditional probability of the event 'the die shows a number greater than 3'given that 'there is at least one head'. **Ans:**$\frac{1}{3}$

179. How many times must a man toss a fair coin so that the probability of having at least one head is more than 90% ? **Ans:**$n = 4$

180. A and B throw a die alternatively till one of them gets a number greater than four and wins the game. If A starts the game, what is the probability of B winning? **Ans: B=**$\frac{2}{5}$, **A=**$\frac{3}{5}$

181. A man is known to speak truth 3 out of 5 times. He throws a die and reports that it is 4. Find the probability that it is actually a 4. **Ans:** $\frac{3}{13}$

182. There are three coins. One is a two – headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the times and third is also a biased coin that comes up tails 40% of the times. One of the three coins is chosen at random and tossed, and it shows head. What is the probability that it was the two – headed coin? **Ans:** $\frac{20}{47}$

183. Two numbers are selected at random (without replacement) from the first six positive integers. Let X denotes the larger of the two numbers obtained. Find the probability distribution of the random variable X, and hence find the mean of the distribution. **Ans:** $\frac{14}{3}$

184. Of people having HIV, 90% of the tests detect the disease but 10% go undetected. Of people free of HIV, 99% of the test judged HIV –ve but 1% are diagnosed as showing HIV +ve . Form a large population of which 0.1% have HIV, one person is selected at random, given the HIV test and the pathologist reports him/her as HIV +ve. What is the probability that the person has actually HIV? What is the importance of conducting awareness programme regarding HIV? **Ans:** 0.0826

185. Assume that the chances of a patient having a heart attack is 40 %. It is also assumed that a meditation and yoga reduce risk of heart attack by 30 % and prescription of certain drug reduces its chances by 25%. at a time a patient can choose any of the two options with equal probabilities. It is given that after going through one of the two options, the patient selected at random suffers heart attack. Find the probability that the patient followed a course of meditation and yoga? What value do you get from it? Is yoga and meditation useful to be free from heart attack? **Ans:** $\frac{14}{29}$

186. A problem in Mathematics is given to 4 students A, B, C, D. Their chances of solving the problem, respectively, are $\frac{1}{3}$. $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{2}{3}$. What is the probability that (i) the problem will be solved? (ii) at most one of them will solve the problem? Ans: $\frac{13}{15}$, $\frac{49}{90}$

187. In a class, 5% of the boys and 10% of the girls have an IQ of more than 150.In the class 60% of the students are boys and rest girls. If a student is selected at random and found to have an IQ of more than 150, find the probability that the selected student is a boy. **Ans:** $\frac{9}{17}$

188. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the next six throw of the die. **Ans:** $\frac{625}{23328}$

189. A bag contains four balls. Two balls are drawn from the bag and found to be white. Find the probability that all balls are white. **Ans :** $\frac{3}{5}$

190. Two cards are drawn successively with replacement from a well- shuffled pack of 52 playing cards. Find the probability distribution of number of kings and hence find the mean of the distribution. **Ans:** $\frac{26}{169}$

191. Two cards are drawn simultaneously (without replacement) from a well – shuffled pack of 52 playing cards. Find the mean, variance and standard deviation of the number of aces. **Ans:** $\frac{34}{221}$

192. A die is thrown 6 times. If 'getting an odd number' is a success. What is the probability of (i) 5 successes? (ii) at least 5 successes? (iii) at most 5 successes? **Ans:** $\frac{3}{32}, \frac{7}{64}, \frac{63}{64}$

193. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$, if he comes by train, bus, and scooter respectively, but if he comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that he comes by train? **Ans:** $\frac{1}{2}$

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194. Show that the vectors If $\vec{a}$, $\vec{b}$, $\vec{c}$ are coplanar, if and only if $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are coplanar

195. If $\vec{a}+\vec{b}+\vec{c}=0$, then prove that $(\vec{a}\times\vec{b})=(\vec{b}\times\vec{c})=(\vec{c}\times\vec{a})$

196. The magnitude of the vector product of the vector $\hat{\imath}+\hat{\jmath}+\hat{k}$ with a unit vector along the sum of the vectors $2\hat{\imath}+4\hat{\jmath}-5\hat{k}$ and $\vec{c}=\lambda\hat{\imath}+2\hat{\jmath}+3\hat{k}$ is equal to $\sqrt{2}$. Find the value of λ. **Ans:** $\lambda = 1$

197. The two adjacent sides of a parallelogram are $2\hat{\imath}-4\hat{\jmath}+5\hat{k}$ and $\hat{\imath}-2\hat{\jmath}-3\hat{k}$. Find the unit vector parallel to one of its diagonals. Also, find its area. **Ans:** $\frac{1}{7}(3\hat{\imath}-6\hat{\jmath}+2\hat{k})$ $11\sqrt{5}$ **sq. unit**

198. Integrate the following w.r.t. x : $\frac{x^2-3x+1}{\sqrt{1-x^2}}$ **Ans:** $\frac{3}{2}\sin^{-1}x-\frac{1}{2}x\sqrt{1-x^2}+3\sqrt{1-x^2}+c$

199. Evaluate: $\int_{-\pi}^{\pi}(\cos a x-\sin b x)^2 \quad dx$ **Ans:** $2\pi+\frac{\sin 2a\pi}{2a}-\frac{\cos 2b\pi}{2b}$

200. If $\vec{r}=x\hat{\imath}+y\hat{\jmath}+z\hat{k}$, find $(\vec{r}\times\hat{\imath}).(\vec{r}\times\hat{\jmath})+xy$ **Ans : 0**

201. If $(tan^{-1}x)^2+(cot^{-1}x)^2=\frac{5\pi^2}{8}$, then find x **Ans: -1**

202. If $x=a\cos\theta+b\sin\theta$, $y=a\sin\theta-b\cos\theta$,show that $y^2\frac{d^2y}{dx^2}-x\ \frac{dy}{dx}+\ y=0$

203. If A and B are two independent events such that P $(\bar{A}\cap B)=\frac{2}{15}$ and P $(A\cap\bar{B})=\frac{1}{6}$, then find P(A) and P(B) **Ans:** $P(A)=\frac{5}{6}$ ***and*** $P(B)=\frac{4}{5}$ **OR :** $P(A)=\frac{1}{5}$ ***and*** $P(B)=\frac{1}{6}$

204. A bag A contains 4 black and 6 red balls and bag B contains 7 black and 3 red balls. A die is thrown. If 1 or 2 appears on it, then bag A is chosen, otherwise bag B. If two balls are drawn at random (without replacement) from the selected bag, Find the probability of one of them being red and the another black. **Ans:** $\frac{22}{45}$

205. Find the local maxima and local minima, of the function $f(x)=\sin x-\cos x$, $0<x<2\pi$ .Also find the local maximum and local minimum values. **Ans: local maxim at** $x=\frac{3\pi}{4}$**,**$\sqrt{2}$**; local minima** $x=\frac{7\pi}{4}$**,**$-\sqrt{2}$

206. Using integration, find the area of the triangle formed by positive x-axis and tangent and the normal to the circle $x^2+y^2=4$ at the point$(1,\ \sqrt{3})$. **Ans:** $2\sqrt{3}$ **sq units**

207. If $(ax+b)e^{\frac{y}{x}}=x$, then show that $x^3\left(\frac{d^2y}{dx^2}\right)=\left(x\frac{dy}{dx}-y\right)^2$

208. If the area bounded by the parabola $y^2=16ax$ and the line $y=4\,m\,x$ is $\frac{a^2}{12}$ sq. units, then using integration, find the value of m. **Ans:** $m=2\sqrt{2}$

209. If $f(x)=\sqrt{x^2+1}$, $g(x)=\frac{x+1}{x^2+1}$ and $h(x)=2x-3$, then find $f'[h'\{g'(x)\}]$ **Ans:** $\frac{2\sqrt{5}}{5}$

210. Find : $\int_0^{\frac{\pi}{4}} \frac{dx}{cos^3 x\sqrt{2\sin 2x}}$ **Ans:** $\frac{6}{5}$

211. Find: $\int \frac{\log x}{(x+1)^2} dx$ **Ans:** $-\frac{\log x}{x+1} + \log\left|\frac{x}{x+1}\right| + c$

212. Prove the following : $\cot^{-1}\left(\frac{xy+1}{x-y}\right) + \cot^{-1}\left(\frac{yz-1}{y-z}\right) + \cot^{-1}\left(\frac{zx+1}{z-x}\right) = 0$

213. Find the value of p for which the curves $x^2 = 9\,p\,(9-y)$ and $x^2 = p\,(y+1)$ cut each other at right angles. **Ans: P=4**

214. Prove the following : $\sin\left[\tan^{-1}\left(\frac{1-x^2}{2\,x}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right] = 1$

215. Find: $\int \frac{dx}{x^3(x^5+1)^{\frac{3}{5}}}$ **Ans:** $-\frac{1}{2}\left(1+\frac{1}{x^5}\right)^{\frac{2}{5}} + c$

216. An unbiased coin is tossed 'n' times . Let the random variable X denote the number of times the head occurs. If P (X = 1), P (X = 2) and P (X = 3) are in AP, find the value of n. **Ans: n = 7**

217. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{1+e^x}\, dx$ **Ans: 1**

218. The two vectors $\hat{j} + \hat{k}$ and $3\,\hat{\imath} - \hat{j} + 4\,\hat{k}$ represent the two side vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ respectively of triangle ABC. Find the length of median through A. **Ans:** $\frac{\sqrt{34}}{2}$ **units**

219. Find the equation of the plane which passes through the point $(3, 2, 0)$ and contains the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$ **Ans:** $x - y + z - 1 = 0$

220. If $\tan^{-1}\left(\frac{1}{1+1.2}\right) + \tan^{-1}\left(\frac{1}{1+2.3}\right) + \ldots\ldots\ldots\ldots + \tan^{-1}\left(\frac{1}{1+n(n+1)}\right) = \tan^{-1}\theta$,then find the value of θ

Ans: $\theta = \frac{n}{n+2}$

221. Find the point on the curve $9\,y^2 = x^3$, where the normal to the curve makes equal intercepts on the axes. **Ans:** $\left(4, \frac{8}{3}\right)$ ***and*** $\left(4, -\frac{8}{3}\right)$

222. If $y = \left(x + \sqrt{1+x^2}\right)^n$, then show that $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n^2 y$

223. Evaluate: $\int e^{2x}.\sin(3x+1)dx$ **Ans:** $\frac{2\,e^{2x}.\sin(3x+1)}{13} - \frac{3\,e^{2x}.\cos(3x+1)}{13} + C$

224. Using integration, find the area of the region bounded by the line $x - y + 2 = 0$ and the curve $x = \sqrt{y}$ and y-axis **Ans:** $\frac{10}{3}$ **sq units**

225. Find the minimum value of $(a\,x + \text{by})$. where $xy = c^2$ **Ans:** $2\sqrt{ab}\,c$

226. Find the co-ordinates of a point of the parabola $y = x^2 + 7x + 2$ which is closest to the straight line $y = 3\,x - 3$ **Ans:** $(-2, -8)$

227. Solve the following differential equation: $\left(\sqrt{1+x^2+y^2+x^2y^2}\right)dx + xy\,dy = 0$.

Ans: $y^2 - 2x^2\cos\left(\frac{y}{x}\right) = c$

228. Find the interval in which the function f given by $f(x) = x^3 + \frac{1}{x^3}$, $x \neq 0$ is (i) increasing (ii) decreasing **Ans: increasing on** $(-\infty, -1) \cup (1, \infty)$; **decreasing on** $(-1, 0) \cup (0, 1)$

229. Prove that $f(\theta) = \frac{4\sin\theta}{2+\cos\theta} - \theta$ **is an increasing function θ in** $\left[0, \frac{\pi}{2}\right]$

230. Find the intervals in which the function f given by $f(x) = \log(1+x) - \frac{2\,x}{2+x}$, is (i) increasing (ii) decreasing **Ans: increasing on** $(-1, \infty)$ **i. e thruogh out its domain**

231. If lines: $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, find the value of k and hence find the equation of the plane containing these lines. **Ans:** $k = \frac{9}{2}$, $5x - 2y - z - 6 = 0$

232. Let N denote the set of all natural numbers and R be the relation on $N \times N$ defined by (a, b) R (c, d) iff $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation.

233. Tangent to the circle $x^2 + y^2 = 4$ at any point on it in the first quadrant makes intercepts OA and OB on x and y axes respectively, O being the centre of the circle find the minimum value of $(OA + OB)$. **Ans: $4\sqrt{2}$ units**

234. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\cos^2 x\, dx}{1+3\sin^2 x}$ **Ans:** $\boldsymbol{\frac{\pi}{6}}$

235. Evaluate: $\int_0^{\frac{\pi}{4}} \frac{\sin x+\cos x}{3+\sin 2x}\, dx$ **Ans:** $\boldsymbol{\frac{1}{4}\log 3}$

236. A man takes a step forward with probability 0.4 and backward with probability 0.6. Find the probability that at the end of 5 steps, he is one step away from the starting point. **Ans:** $\boldsymbol{\frac{72}{125}}$

237. Find the absolute maximum and absolute minimum values of the function f given by $f(x) = \sin^2 x - \cos x,\ x \in [0, \pi]$ **Ans: absolute maximum= $\frac{5}{4}$, absolute minimum= -1**

238. Find the absolute maximum and absolute minimum values of the function f given by $f(x) = \cos^2 x + \sin x,\ x \in [0, \pi]$ **Ans: absolute maximum= $\frac{5}{4}$ at $x = \frac{\pi}{6}$ and $\frac{5\pi}{6}$, absolute minimum= 1 at $x = 0, \frac{\pi}{2}$ *and* π**

239. Consider a function $f : R_+ \to [-9, \infty)$ given by $f(x) = 5x^2 + 6x - 9$, prove that that f is invertible with $\mathbf{f^{-1}(y)} = \frac{\sqrt{54+5y}-3}{5}$

240. A bag contains $(2n + 1)$ coins. It is known that $(n - 1)$ of these coins have a head on both sides, whereas the rest of the coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is $\frac{31}{42}$, determine the value of n. Ans: n = 31

241. Find the equation of tangents to the curve $y = \cos(x + y),\ -2\pi \le x \le 2\pi$ that are parallel to the line $x + 2y = 0$ **Ans: $2x + 4y + 3\pi = 0$ or $2x + 4y - \pi = 0$**

242. Show that the function f given by $f(x) = \tan^{-1}(\sin x + \cos x),\ x > 0$ is always an strictly increasing function in $\left(0, \frac{\pi}{4}\right)$.

243. Find the points of local maxima, local minima and the points of inflection of the function $f(x) = x^5 - 5x^4 + 5x^3 - 1$. Also, find the corresponding local maximum and local minimum values. **Ans: local maxim at $x = 1$, *value*: 0 ; local minima $x = 3$; *value*: −28, Point of inflection $x = 0$**

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